

Pilgrim Dark Energy

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ABSTRACT

In the present work, we reconsider the idea of holographic dark energy. One of its key points is the formation of the black hole. And then, we propose the so-called “pilgrim dark energy” based on the speculation that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. We also consider the cosmological constraints on pilgrim dark energy by using the latest observational data. Of course, one can instead regard pilgrim dark energy as a purely phenomenological model without any physical motivation. We also briefly discuss this issue.

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I. INTRODUCTION

Since the discovery of the current accelerated expansion of our universe, dark energy has been one of the most active fields in physics and astronomy [1]. Of course, the simplest candidate of dark energy is a tiny positive cosmological constant, Λ . However, as is well known, it is plagued with the cosmological constant problem (the fine-tuning problem) and the cosmological coincidence problem. Therefore, many alternative dark energy models have been proposed.

Recently, the so-called holographic dark energy (HDE) has been considered as an interesting candidate of dark energy, which has been studied extensively in the literature. For a quantum gravity system, the local quantum field cannot contain too many degrees of freedom, otherwise the black hole forms and then the quantum field theory breaks down. In the black hole thermodynamics [2, 3], there is a maximum entropy in a box of size L , namely the Bekenstein-Hawking entropy bound S_{BH} , which scales as the area of the box $\sim L^2$, rather than the volume $\sim L^3$. To avoid the breakdown of the local quantum field theory, Cohen *et al.* [4] proposed the so-called energy bound, which is more restrictive than the entropy bound. If ρ_Λ is the quantum zero-point energy density caused by a short distance cut-off, the total energy in a box of size L cannot exceed the mass of a black hole of the same size [4], namely $\rho_\Lambda L^3 \lesssim m_p^2 L$, where $m_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. The largest IR cut-off L is the one saturating the inequality. Therefore, one has

$$\rho_\Lambda = 3c^2 m_p^2 L^{-2}, \quad (1)$$

where the numerical constant $3c^2$ is introduced for convenience. In the literature, many IR cut-offs L have been considered, such as the Hubble horizon H^{-1} [5, 6], the particle horizon $R_H \equiv a \int_0^t dt/a = a \int_0^a d\tilde{a}/(H\tilde{a}^2)$ [7, 43], the future event horizon $R_h \equiv a \int_t^\infty dt/a = a \int_a^\infty d\tilde{a}/(H\tilde{a}^2)$ [8], the Ricci scalar curvature radius, which is actually proportional to the causal connection scale of perturbations in the flat universe $R_{CC} = (\dot{H} + 2H^2)^{-1/2}$ [9], the formal generalization of R_{CC} , namely $(\alpha H^2 + \beta \dot{H})^{-1/2}$ [10], the age of our universe $T = \int_0^a d\tilde{a}/(H\tilde{a})$ [11], the conformal age of our universe $\eta \equiv \int_0^t dt/a = \int_0^a d\tilde{a}/(\tilde{a}^2 H)$ [12], the radius of the cosmic null hypersurface [13], the so-called conformal-age-like length [14], where $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1+z)^{-1}$ is the scale factor of our universe; z is the redshift; we have set $a_0 = 1$; the subscript “0” indicates the present value of the corresponding quantity; a dot denotes the derivative with respect to cosmic time t .

In the present work, we reconsider the idea of holographic dark energy. One of its key points is the formation of the black hole. If we can prevent the formation of the black hole, the energy bound proposed by Cohen *et al.* [4] could be violated. If the repulsive force is strong enough, it might resist the matter collapse and then black hole does not form. So, what can contribute the strong repulsive force? Nowadays, it is well known that dark energy can contribute a repulsive force, since its equation-of-state parameter (EoS) $w < -1/3$. However, we will see below that the repulsive force contributed by the quintessence-like dark energy ($w > -1$) is not strong enough to prevent the formation of the black hole. Therefore, we focus on the phantom-like dark energy ($w < -1$). Firstly, it is well known that everything will be completely torn up before our universe ends in the big rip caused by the phantom-like dark energy. Even the black hole will also be completely torn up. So, one can find that the repulsive force contributed by the phantom-like dark energy is strong enough to destroy the black hole. Secondly, in the famous work by Babichev *et al.* [15], it is shown that accretion of phantom-like dark energy is accompanied with the gradual decrease of the black hole mass. Of course, this issue has not been completely settled by now. In [16], Gao *et al.* claimed that the physical black hole mass may instead increase due to the accretion of phantom energy, but the Cosmic Censorship Conjecture will be violated. Later, Gonzalez and Guzman [17] presented the full non-linear study of phantom scalar field accreted into a black hole, and found that the accretion of the phantom scalar field into the black hole can reduce its area down to 50% within time scales of the order of few masses of the initial horizon. In [18], Sun also claimed that in the phantom dark energy universe the black hole mass becomes zero before the big rip is reached. On the other hand, Jamil and Qadir [19] claimed that the phantom energy accretion contribute to a decrease of the mass of the primordial black hole. Recently, Sharif and Abbas [20] also found that mass of the black hole decreases due to phantom accretion. Thirdly, Harada *et al.* [21] claimed that there is no self-similar black hole solution in an universe with a stiff fluid or scalar field or quintessence. It is worth noting that Akhoury *et al.* [44] independently obtained the similar result from a very different

perspective. Chapline [22] claimed that black holes might not exist in our real world, because the negative pressure contributed by dark energy might prevent black holes from forming. Note that in [21, 22, 44] they only require dark energy violating the strong energy condition. Of course, they are wrong, because the repulsive force contributed by the quintessence-like dark energy ($w > -1$) is not strong enough to prevent the formation of the black hole. This point can be seen in [23], where by considering some specific McVittie solutions, Li and Wang showed that black holes can exist in a Friedmann-Robertson-Walker (FRW) universe dominated by dark energy. However, they require that the weak energy condition should be satisfied (although the other three energy conditions can be violated). Here, we would like to stress that the phantom-like dark energy does not satisfy the weak energy condition (in fact it violates all energy conditions). Therefore, the conclusion of Li and Wang [23] cannot be applied to the phantom-like dark energy ($w < -1$). On the other hand, Rahaman *et al.* [24] claimed that they have found an exact solution of spherically symmetrical Einstein equations describing a black hole with a special type “phantom” energy source. Unfortunately, this special type “phantom” energy source used in [24] is characterized by $p = -\rho$ in fact. Obviously, it is not the commonly called phantom energy ($p < -\rho$), and hence the conclusion of [24] cannot be applied here.

Together with these three arguments mentioned above, it is reasonable to speculate that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. Of course, we admit that this speculation need further and solid proofs which are still absent. Anyway, we suggest considering what will happen to the argument of holographic dark energy if this speculation is true. In Sec. II, we propose the so-called “pilgrim dark energy” based on this speculation. Then, in Sec. III, we consider the cosmological constraints on pilgrim dark energy by using the latest observational data. Some concluding remarks are given in Sec. IV.

II. PILGRIM DARK ENERGY

Here, we propose the so-called pilgrim dark energy (PDE) based on the aforementioned speculation that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. If this speculation is true, the energy bound proposed by Cohen *et al.* [4] could be violated, namely the total energy in a box of size L could exceed the mass of a black hole of the same size [4], i.e., $\rho_\Lambda L^3 \gtrsim m_p^2 L$. Therefore, the first property of pilgrim dark energy is

$$\rho_\Lambda \gtrsim m_p^2 L^{-2}. \quad (2)$$

To implement Eq. (2), the simplest way is to set

$$\rho_\Lambda = 3n^2 m_p^{4-s} L^{-s}, \quad (3)$$

where n and s are both dimensionless constants. Thus, from Eqs. (2) and (3), $\rho_\Lambda \sim m_p^{4-s} L^{-s} \gtrsim m_p^2 L^{-2}$ leads to $L^{2-s} \gtrsim m_p^{s-2} = \ell_p^{2-s}$, where $\ell_p = m_p^{-1} = 1.616 \times 10^{-33}$ cm is the reduced Planck length, which is extremely short length in fact. Obviously, since $L > \ell_p$ in general, it is required that

$$s \leq 2. \quad (4)$$

As mentioned above, the second requirement of pilgrim dark energy is to be phantom-like, namely

$$w_\Lambda < -1. \quad (5)$$

Here, in order to obtain the EoS of pilgrim dark energy, we should choose a specific cut-off L . Of course, the simplest choice is the Hubble horizon $L = H^{-1}$. In the epoch dominated by pilgrim dark energy, the Friedmann equation $H^2 \rightarrow \rho_\Lambda / (3m_p^2) = n^2 m_p^{2-s} H^s$ leads to $H \rightarrow \text{const.}$, which corresponds to $w_\Lambda \rightarrow -1$. Therefore, our universe will end in a de Sitter phase, rather than big rip. In the matter-dominated epoch, $H^2 \sim \rho_m \sim a^{-3}$. So, $\rho_\Lambda \sim H^s \sim a^{-3s/2}$. Thus, we have $w_\Lambda = -1 + s/2$. From Eq. (5), $s < 0$ is required. Let us consider the general case. Substituting $\rho_\Lambda \propto H^s$ into the energy conservation equation $\dot{\rho}_\Lambda + 3H\rho_\Lambda(1 + w_\Lambda) = 0$, we have

$$w_\Lambda = -1 - \frac{s\dot{H}}{3H^2}. \quad (6)$$

One might naively consider that $\dot{H} > 0$ in the late time, since $w_\Lambda < -1$. However, it is wrong. The total EoS $w_{tot} = \Omega_\Lambda w_\Lambda \rightarrow -1$, where Ω_Λ is the fractional energy density of pilgrim dark energy. Therefore, $\dot{H} < 0$ instead holds in the late time. If $\dot{H} < 0$ is valid in the whole cosmic history, from Eqs. (5) and (6), it is required that

$$s < 0. \quad (7)$$

So, there is no conflict between the matter-dominated epoch and the late time. w_Λ starts from $-1 + s/2$ in the matter-dominated epoch and goes asymptotically to -1 in the late time; $w_\Lambda < -1$ always and it never crosses the phantom divide $w = -1$ in the whole cosmic history. To be clearer, let us see a particular example of $s = -2$. In this case, substituting $\rho_m = \rho_{m0}a^{-3}$ and Eq. (3) with $L = H^{-1}$ into the Friedmann equation, we obtain

$$H^2 = \frac{1}{3m_p^2}(\rho_\Lambda + \rho_m) = n^2 m_p^4 H^{-2} + \frac{\rho_{m0}}{3m_p^2} a^{-3} \equiv c_1 H^{-2} + c_2 a^{-3}. \quad (8)$$

Note that we consider a flat FRW universe which only contains pressureless matter and dark energy throughout this work. Requiring $H^2 \geq 0$, we can solve Eq. (8) and obtain

$$H^2 = \frac{1}{2} \left(c_2 a^{-3} + \sqrt{4c_1 + c_2^2 a^{-6}} \right). \quad (9)$$

When $a \rightarrow 0$, $H^2 \rightarrow c_2 a^{-3}$, which coincides with the one of matter-dominated epoch. When $a \rightarrow \infty$, $H^2 \rightarrow \sqrt{c_1} = \text{const.}$, which coincides with the one of de Sitter phase in the late time. From Eq. (9), it is easy to see that H decreases when a increases; H is a monotonically decreasing function of a . Thus, $\dot{H} < 0$ indeed holds in the whole cosmic history. So, if $s < 0$, we have $w_\Lambda < -1$ in the whole cosmic history. Note that if $s = 0$, from Eq. (3), it is easy to see that our pilgrim dark energy model reduces to the well-known Λ CDM model. Considering this point as well as Eqs. (4) and (7), we allow

$$s \leq 0, \quad (10)$$

in order to satisfy the requirements (2) and (5), and include Λ CDM as a special case (although $w_\Lambda = -1$ in this case).

At first glance, one might consider that pilgrim dark energy model is a three-parameter model, which contains n , s and Ω_{m0} as model parameters (where Ω_{m0} is the present fractional energy density of pressureless matter). However, one of them is not independent in fact. Substituting $\rho_m = \rho_{m0}a^{-3}$ and Eq. (3) with $L = H^{-1}$ into the Friedmann equation, namely $3m_p^2 H^2 = \rho_\Lambda + \rho_m$, we have

$$n^2 m_p^{2-s} H^{s-2} + \frac{\rho_{m0}}{3m_p^2 H^2} a^{-3} = 1. \quad (11)$$

Introducing dimensionless $E \equiv H/H_0$ and $\tilde{n}^2 \equiv n^2(m_p/H_0)^{2-s}$, we recast Eq. (11) as

$$\tilde{n}^2 E^{s-2} + \Omega_{m0} E^{-2} a^{-3} = 1. \quad (12)$$

Requiring $E(a=1) = 1$ by definition, we find that

$$\tilde{n}^2 = 1 - \Omega_{m0}. \quad (13)$$

So, n (or equivalently \tilde{n}) is not independent. Finally, the Friedmann equation becomes

$$(1 - \Omega_{m0}) E^{s-2} + \Omega_{m0} E^{-2} (1+z)^3 = 1. \quad (14)$$

Obviously, there are only two free model parameters, namely Ω_{m0} and s . From Eq. (14), one can obtain $E(z)$ as a function of redshift z , if model parameters Ω_{m0} and s are given. Note that E is a real number and $E \geq 0$ is required by definition.

Here, we would like to say some words before going further. As is shown above, pilgrim dark energy is based on the aforementioned speculation that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. In fact, this speculation is proposed just from some arguments, and it has no solid foundation so far. If one cannot agree the physical motivation of pilgrim dark energy presented here, we suggest that one can instead regard pilgrim dark energy as a purely phenomenological model without invoking any physical motivation.

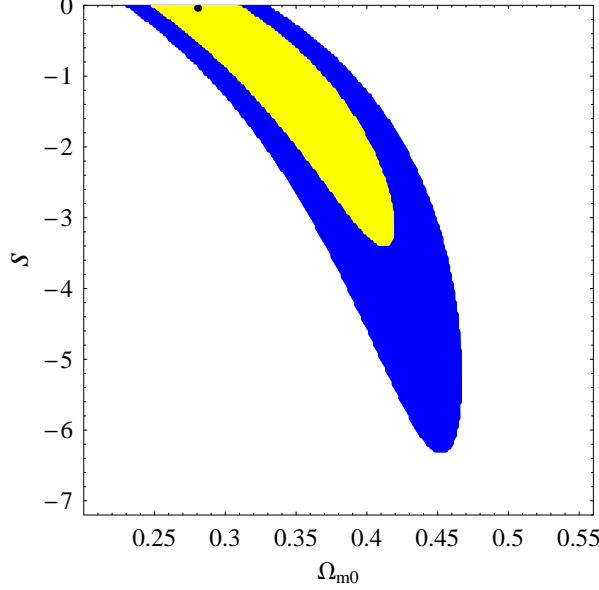


FIG. 1: The 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - s$ parameter space. The best-fit parameters are also indicated by a black solid point. This result is obtained by using the data of 580 Union2.1 SNIa alone.

III. COSMOLOGICAL CONSTRAINTS ON PILGRIM DARK ENERGY

In this section, we consider the cosmological constraints on pilgrim dark energy by using the latest observational data.

At first, we use the observational data of Type Ia supernovae (SNIa) alone. Recently, the Supernova Cosmology Project (SCP) collaboration released the updated Union2.1 compilation which consists of 580 SNIa [25]. The Union2.1 compilation is the largest published and spectroscopically confirmed SNIa sample to date. The data points of the 580 Union2.1 SNIa compiled in [25] are given in terms of the distance modulus $\mu_{obs}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{th}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (15)$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and h is the Hubble constant H_0 in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1+z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z}; \mathbf{p})}, \quad (16)$$

in which \mathbf{p} denotes the model parameters. The χ^2 from 580 Union2.1 SNIa is given by

$$\chi_\mu^2(\mathbf{p}) = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma^2(z_i)}, \quad (17)$$

where σ is the corresponding 1σ error. The parameter μ_0 is a nuisance parameter, but it is independent of the data points. One can perform a uniform marginalization over μ_0 . However, there is an alternative way. Following [26, 27], the minimization with respect to μ_0 can be made by expanding the χ_μ^2 of Eq. (17) with respect to μ_0 as

$$\chi_\mu^2(\mathbf{p}) = \tilde{A} - 2\mu_0 \tilde{B} + \mu_0^2 \tilde{C}, \quad (18)$$

where

$$\tilde{A}(\mathbf{p}) = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \mathbf{p})]^2}{\sigma_{\mu_{obs}}^2(z_i)},$$

$$\tilde{B}(\mathbf{p}) = \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \mathbf{p})}{\sigma_{\mu_{obs}}^2(z_i)}, \quad \tilde{C} = \sum_i \frac{1}{\sigma_{\mu_{obs}}^2(z_i)}.$$

Eq. (18) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}_\mu^2(\mathbf{p}) = \tilde{A}(\mathbf{p}) - \frac{\tilde{B}(\mathbf{p})^2}{\tilde{C}}. \quad (19)$$

Since $\chi_{\mu, min}^2 = \tilde{\chi}_{\mu, min}^2$ obviously (up to a constant), we can instead minimize $\tilde{\chi}_\mu^2$ which is independent of μ_0 . The best-fit model parameters are determined by minimizing the total χ^2 . When SNIa is used alone, we have $\chi^2 = \tilde{\chi}_\mu^2$ which is given in Eq. (19). As in [27, 28], the 68.3% confidence level (C.L.) is determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2 \leq 1.0, 2.3$ and 3.53 for $n_p = 1, 2$ and 3 , respectively, where n_p is the number of free model parameters. Similarly, the 95.4% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2 \leq 4.0, 6.17$ and 8.02 for $n_p = 1, 2$ and 3 , respectively. Here, we scan the $\Omega_{m0} - s$ parameter space (note that as mentioned above, $s \leq 0$ and $0 \leq \Omega_{m0} \leq 1$ are required), and solve Eq. (14) to obtain $E(z)$ as a function of redshift z . Therefore, the corresponding χ^2 is on hand. Finally, we find that the best fit has $\chi_{min}^2 = 562.226$, and the corresponding best-fit parameters are $\Omega_{m0} = 0.280$ and $s = -0.04$. In Fig. 1, we present the corresponding 68.3% and 95.4% C.L. contours in the $\Omega_{m0} - s$ parameter space. It is easy to see that although the best-fit s is close to zero, there is a very big room for a significantly non-zero s in the 95.4% C.L. region. In fact, the viable s can extend to about -6.4 at 95.4% C.L., or -3.4 at 68.3% C.L. The price to have a significantly non-zero s is a larger Ω_{m0} .

Next, we add the data from the observation of the large-scale structure (LSS). Here we use the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies [29, 30], which contains the main information of the observations of LSS. The distance parameter A is given by

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3}, \quad (20)$$

where $z_b = 0.35$. In [30], the value of A has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here the scalar spectral index n_s is taken to be 0.963, which has been updated from the WMAP 7-year (WMAP7)

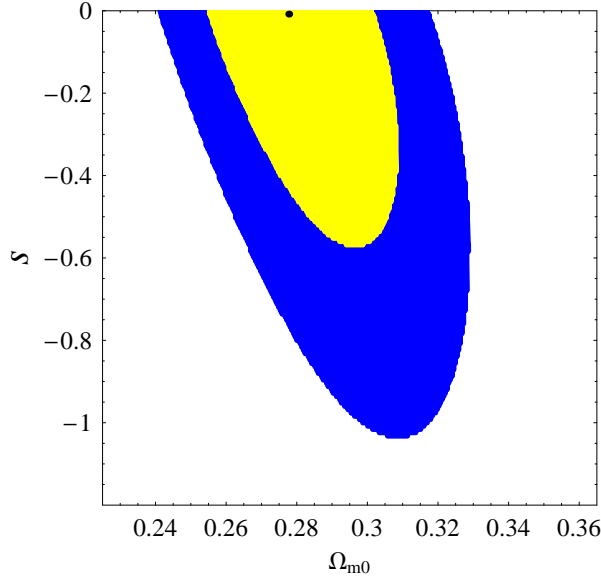


FIG. 2: The same as in Fig. 1, except that this result is obtained by using the combined observational data of 580 Union2.1 SNIa and the distance parameter A from BAO.

data [31]. Now, the total $\chi^2 = \tilde{\chi}_\mu^2 + \chi_A^2$, where $\tilde{\chi}_\mu^2$ is given in Eq. (19), and $\chi_A^2 = (A - A_{obs})^2/\sigma_A^2$. Again, we scan the $\Omega_{m0} - s$ parameter space, and find that the best fit has $\chi_{min}^2 = 562.227$, and the corresponding best-fit parameters are $\Omega_{m0} = 0.278$ and $s = -0.008$. In Fig. 2, we present the corresponding 68.3% and 95.4% C.L. contours in the $\Omega_{m0} - s$ parameter space. Comparing with Fig. 1, it is easy to see that the contours are significantly shrunk. Although the best-fit s is very close to zero, there is still a room for a significantly non-zero s in the 95.4% C.L. region. In fact, the viable s can extend to about -1.05 at 95.4% C.L., or -0.6 at 68.3% C.L.

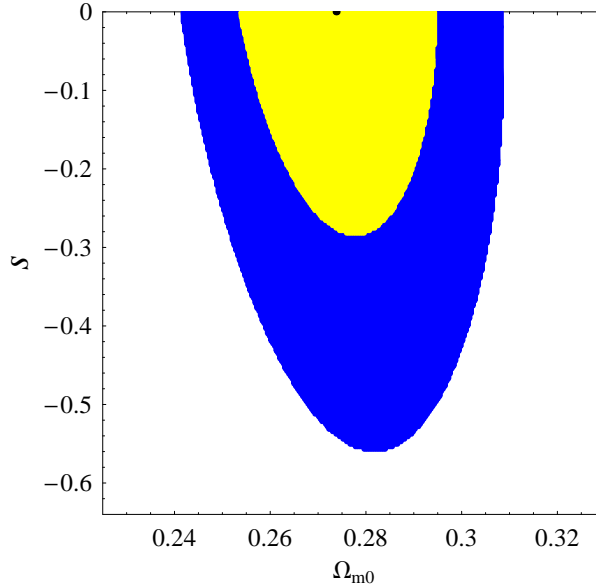


FIG. 3: The same as in Fig. 1, except that this result is obtained by using the combined observational data of 580 Union2.1 SNIa, the distance parameter A from BAO, and the shift parameter R from WMAP7.

Finally, we further add the data from the observation of the cosmic microwave background (CMB). Here we use the the shift parameter R , which contains the main information of the observations of the CMB [31–33]. The shift parameter R of the CMB is defined by [32, 33]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_*} \frac{d\tilde{z}}{E(\tilde{z})}, \quad (21)$$

where the redshift of recombination $z_* = 1091.3$ which has been updated in the WMAP7 data [31]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [32, 33]. The value of R has been updated to 1.725 ± 0.018 from the WMAP7 data [31]. Now, the total $\chi^2 = \tilde{\chi}_\mu^2 + \chi_A^2 + \chi_R^2$, where $\chi_R^2 = (R - R_{obs})^2/\sigma_R^2$. We scan the $\Omega_{m0} - s$ parameter space, and find that the best fit has $\chi_{min}^2 = 562.546$, and the corresponding best-fit parameters are $\Omega_{m0} = 0.274$ and $s = 0.0$. In Fig. 3, we present the corresponding 68.3% and 95.4% C.L. contours in the $\Omega_{m0} - s$ parameter space. Comparing with Fig. 2, it is easy to see that the contours are further shrunk. Although the best-fit s becomes zero, there is still a room for a non-zero s in the 95.4% C.L. region. In fact, the viable s can extend to about -0.56 at 95.4% C.L., or -0.29 at 68.3% C.L.

IV. CONCLUDING REMARKS

Some remarks are in order. Firstly, as mentioned above, pilgrim dark energy model reduces to the well-known Λ CDM model if $s = 0$. On the other hand, when we constrain pilgrim dark energy by

using the latest observational data, the best-fit parameter s is zero. However, this does not mean that pilgrim dark energy fails, because there is still a room for a non-zero s in the 95.4% C.L. region. For instance, pilgrim dark energy with $s = -1/2$ is still viable. On the other hand, as is well known, Λ CDM model is plagued with the cosmological constant problem (the fine-tuning problem) and the cosmological coincidence problem. Therefore, pilgrim dark energy still deserves further investigations.

Secondly, although $w_\Lambda < -1$ in the whole cosmic history, it will go asymptotically to -1 in the late time. So, our universe will end in a de Sitter phase, rather than big rip. This is an advantage of pilgrim dark energy model.

Thirdly, in the present work, we have chosen IR cut-off $L = H^{-1}$ for pilgrim dark energy, since the Hubble horizon H^{-1} is the simplest case. However, one can instead choose other IR cut-offs L for pilgrim dark energy, such as the particle horizon $R_H \equiv a \int_0^t dt/a = a \int_0^t d\tilde{a}/(H\tilde{a}^2)$ [7], the future event horizon $R_h \equiv a \int_t^\infty dt/a = a \int_a^\infty d\tilde{a}/(H\tilde{a}^2)$ [8], the Ricci scalar curvature radius, which is actually proportional to the causal connection scale of perturbations in the flat universe $R_{CC} = (\dot{H} + 2H^2)^{-1/2}$ [9], the formal generalization of R_{CC} , namely $(\alpha H^2 + \beta \dot{H})^{-1/2}$ [10], the age of our universe $T = \int_0^a d\tilde{a}/(H\tilde{a})$ [11], the conformal age of our universe $\eta \equiv \int_0^t dt/a = \int_0^a d\tilde{a}/(\tilde{a}^2 H)$ [12], the radius of the cosmic null hypersurface [13], the so-called conformal-age-like length [14]. This situation is very similar to holographic dark energy. It is possible to have completely new results (e.g. the cosmological constraints might be different) if L is changed. Of course, we welcome other authors to explore this possibility.

Fourthly, as is shown above, pilgrim dark energy is based on the aforementioned speculation that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. In fact, this speculation is proposed just from some arguments, and it has no solid foundation so far. If one cannot agree the physical motivation of pilgrim dark energy presented here, we suggest that one can instead regard pilgrim dark energy as a purely phenomenological model without invoking any physical motivation.

Finally, if we consider pilgrim dark energy as a purely phenomenological model without any physical motivation, the physical requirement $s \leq 0$ can be given up. So, pilgrim dark energy can have a richer phenomenology. On the other hand, we also note that pilgrim dark energy can include other existing dark energy models as its special cases. For instance, if $s = 1$ and $L = H^{-1}$, pilgrim dark energy reduces to the so-called QCD ghost dark energy [34–37], and Dvali-Gabadadze-Porrati (DGP) model [38, 39]. Of course, if $s = 2$ and choosing various L , pilgrim dark energy reduces to holographic dark energy [8], (new) agegraphic dark energy [11, 12], Ricci dark energy [9], and so on. If $L = H^{-1}$ and s is free, pilgrim dark energy reduces to the modified DGP model (α dark energy) [40], and the modified holographic dark energy with IR infinite extra dimensions [41]. If $L = H^{-1}$, R_H , R_h , and s is free, pilgrim dark energy reduces to the so-called holographic cosmological “constant” derived from a generalized holographic relation between UV and IR cut-offs [42]. We anticipate that pilgrim dark energy with various L and a completely free parameter s (which is not necessary to be $s \leq 0$) is a rich mine.

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